CS 285 Final Project - Design, Modeling and Fabrication of the Möbius Gear

Aaron M. Hoover

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1 Introduction

The purpose of this final project, undertaken for CS285 - Procedural Solid Modeling, was to design, procedurally model, and fabricate a functional Möbius gear using a variety of solid modeling software tools and the solid free form rapid prototyping technologies available in several labs here at Berkeley. The Möbius gear merges the geometric beauty of the Möbius surface form with functional design embodied in the mechanical gear elements. As such, a fully functional physical realization of the Möbius gear poses unique design challenges in both the modeling and fabrication processes. The modeling process requires software tools powerful and flexible enough to generate arbitrary parameterized mathematical surfaces, yet easy enough to use to make the addition of engineering-type structural features straightforward. As we will see later in this report, no such single tool exists and so an ability to move between tools for different tasks is crucial to success.

Figure 1: Graphic rendering of the Möbius gear courtesy of Tom Longtin

As we can clearly see from Fig. 1, the fabrication of the Möbius gear requires that we take advantage of compliant or elastic materials for at least the central Möbius strip in order to allow the strip to twist as it's rotated about the central axis. Readily available rapid prototyping machines here at Berkeley don't have the capability to directly produce elastic parts, so we augment them with addtional processes. Overall, in going from the 2D graphic visualization of the Möbius gear to a working physical artifact, we bring several modeling and solid free form fabrication tools to bear on the problem.

The remainder of this report is structured as follows: the second section details the process of modeling the general form of the Möbius gear using the Scene Language for Interactive Dynamic Environments, or SLIDE. In that section, we discuss specific challenges of procedurally controlling the geometry of the gear "ribbons." we also present a straightforward mathematical parameterization of the shape that aids in the generation of constant pitch gear ribbons despite curvature changes. In the third section, we discuss the process of preparing the model for the rapid prototyping machines. In this section, fabrication details and requirements are explicitly considered and additional modeling tools and strategies are introduced with the specific goal of producing models suitable for production on the Stratasys Fused Deposition Modeling machine in the Ford lab and the 3D Systems ThermoJet wax deposition machine in the Biomimetic Millisystem lab. The fourth section details the actual fabrication process in which the physical artifacts are produced, and the last section is simply a collection of lessons learned and insights gained in the process of completing the project.

2 Modeling in SLIDE

Working from a graphic rendering of the Möbius gear depicted in Fig. 1 we begin by modeling the basic form of the Möbius gear in SLIDE as a 360^o sweep of the cross section pictured in Fig. 2 with a 180^o twist. Modeling the

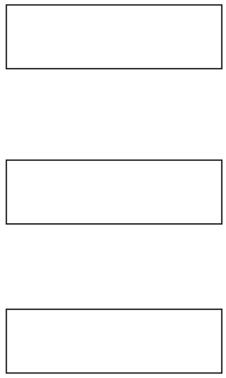

Figure 2: Initial sweep cross section for the Möbius gear

cross section as three separate curves and sweeping over 360^o simplifies the sweep, allowing us to create the inner Möbius ring and the outer ring at the same time, but creates effectively three solid bodies instead of two. Next, control points which will control the tooth height on the inner Möbius strip are placed along the circle representing the sweep path. For each tooth we intend to generate, we place 4 control points in equally spaced increments of θ, or sweep angle, along the sweep path. Each control point has a scale factor associated with it indicating how the cross section should be scaled in each of the three coordinate axes at that point along the sweep. So, by scaling the cross section up and down in the Z direction as the sweep traverses the path, we create undulations on the surface of the inner Möbius strip resembling gear teeth. (It should be noted that SLIDE lacks the functionality to create control points procedurally using Tcl functions, so a Python script is used to generate and insert the appropriate static SLIDE text into a SLIDE template file in order to generate teeth. This has the effect that the number of teeth in the part cannot be adjusted dynamically while SLIDE is running.) The same control point approach is taken for the cross section of the outer ring, with the scale factor calculated appropriately so that a constant distance is maintained between the peaks of the teeth on the outer ring and the troughs on the inner ring. The result is an inner ring with two sides undulating in and outward from the central plane and an outer ring whose entire cross section is scaled in and outward from the sweep path, pictured in Fig. 3.

Careful examination of Fig. 3 shows that simply placing teeth in equal θ increments produces variation in gear pitch on the outer ring that will likely prevent the mechanism from functioning properly.

To solve this potential problem, we construct the outer ring as a single sweep of a closed rectangular curve offset from the inner ring by a fixed distance and swept through an angle of 720^o. Constructing the outer ring in this manner enables us to explicitly parameterize the position of the center of the offset curve entirely in terms of the sweep angle, θ. Once the position is parameterized in terms of θ, we can calculate the derivative of the position with respect to θ to obtain an instantaneous "velocity" at any point along the curve. The next increment in θ at which to place a control point is then computed by scaling the average length, or the total arc length divided by the number of control points, by the inverse of the instantaneous velocity at that particular point. This relationship is summarized in the equations below with Fig. 4 depicting the vector representations of the position of the center of the cross section of the outer ring. In Fig. 4 θ is the sweep angle, ϕ is the the rotation angle of the outer cross section about the inner ring, r is the radius of the inner Möbius ring, and a is the distance the center of the outer ring is offset from the center of the inner ring.

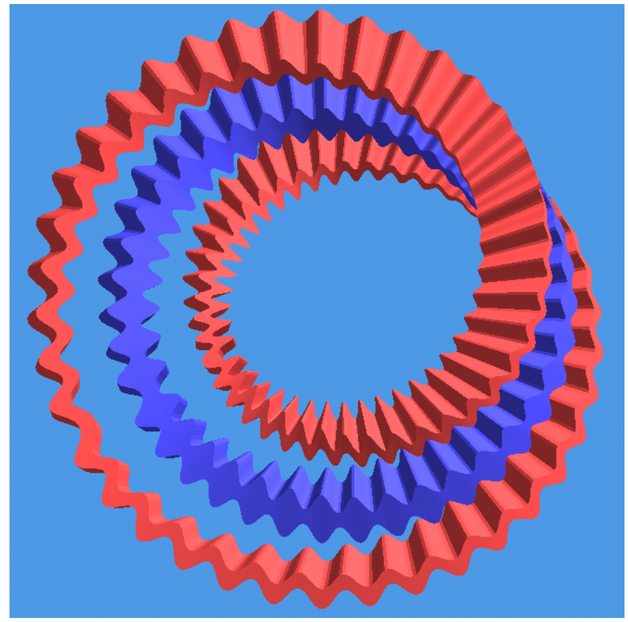

Figure 3: Results of initial gear "tooth" generation

$$\mathbf{p} = r\mathbf{e}_r + a\sin(\phi)\mathbf{e}_r + a\cos(\phi)\mathbf{e}_z \tag{1}$$

$$\mathbf{e}_r = \cos(\theta)\mathbf{e}_1 + \sin(\theta)\mathbf{e}_2 \tag{2}$$

$$\phi = \frac{\theta}{2} \tag{3}$$

$$s = \int_{\theta_1}^{\theta_2} \left|\left|\frac{\partial \mathbf{p}}{\partial \theta}\right|\right| d\theta \tag{4}$$

$$\left|\left|\frac{\partial \mathbf{p}}{\partial \theta}\right|\right| = \sqrt{\frac{3a^2}{4} + r^2 - \frac{a^2\cos(\theta)}{2} - 2ar\sin\left(\frac{\theta}{2}\right)} \tag{5}$$

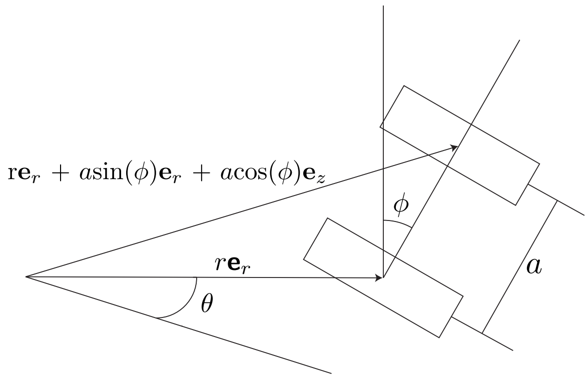

Figure 4: Parameterization of outer ring sweep position

$$s = \int_{\theta_1}^{\theta_2} \sqrt{\frac{3a^2}{4} + r^2 - \frac{a^2\cos(\theta)}{2} - 2ar\sin\left(\frac{\theta}{2}\right)}\, d\theta \quad (6)$$

$$d\theta = \frac{ds}{\sqrt{\frac{3a^2}{4} + r^2 - \frac{a^2\cos(\theta)}{2} - 2ar\sin\left(\frac{\theta}{2}\right)}} \quad (7)$$

Using the above parameterization to compute the steps in θ at which to place control points for generation of "teeth" yields a model which much more closely approximates a constant pitch gear. The results are show in Fig. 5.

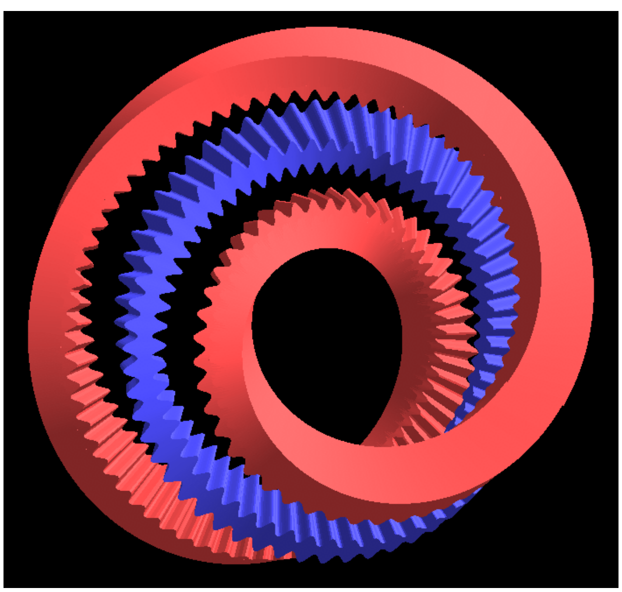

Figure 5: Results of initial gear "tooth" generation

Lastly, a central ridge was trivially added using another sweep to both the inner and outer strips to act as a guide for the gear wheels to constrain them to roll along the curves.

3 Fabrication Preparation

In the initial design stage, it was decided that the outer ring would be rigid and attached to a base, while the inner Möbius strip would rotate. Therefore, the outer ring would be fabricated on the FDM machine. In order to improve the structural integrity of the outer ring, it is necessary to add a base as well as some degree of structural scaffolding. However, adding structural scaffolding procedurally in SLIDE proved to be difficult because adding additional sweeps to the existing file began to cause SLIDE to crash. To solve this problem, the SLIDE file was exported to STL format. The STL was then imported into Solidworks as a graphics body. This option simply renders the STL data in the Solidworks scene but does not attempt to convert the faces of the triangle to Solidworks surfaces. It is, however, still useful because it enables us to use the graphics body as a visual reference. That is, true solid body features can be created in such a way that they intersect the graphics body at points specified by the user. This allows the user to create solid features in rough reference to the geometry of the graphics body, and since both the solid bodies and the graphics body share an origin, they can be easily aligned at a later step in the process. Fig. 6 is a screenshot from Solidworks with the graphics body highlighted in green and the scaffolding solid body in grey.

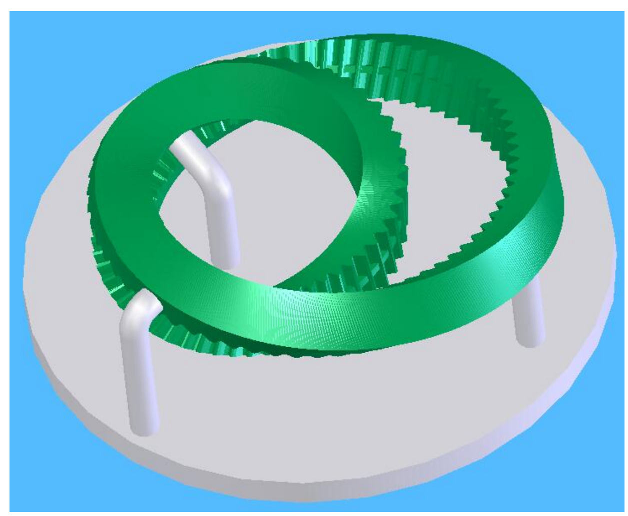

Figure 6: Creation of solid bodies in Solidworks using a graphics body for reference

Finally, the STL file exported from SLIDE containing the outer ring of the Möbius gear and the STL file exported from Solidworks containing the scaffolding are imported into Quickslice, aligned, and joined to form a single STL file shown in Fig. 7.

The last remaining step before fabrication can begin on the FDM machine is slicing the model in Quickslice to

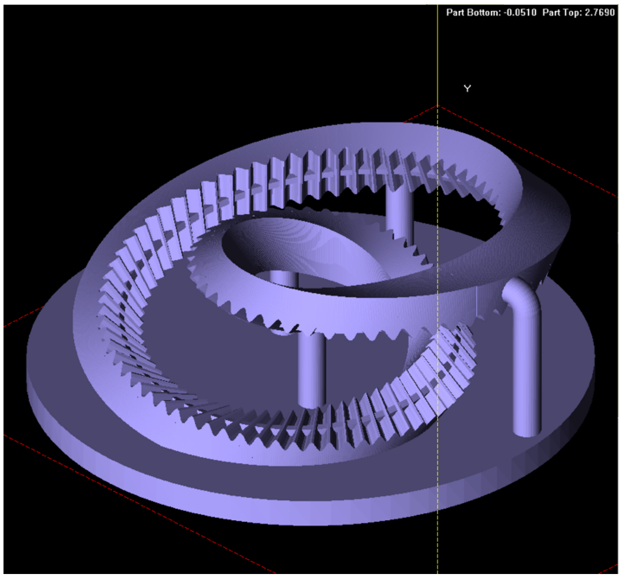

Figure 7: Single STL file created in Quickslice

ensure that the geometry does contain problematic areas for the machine resulting from issues like self-intersecting curves or exceedingly thin walls. The part was sliced several times, a few minor revisions were made in an effort to reduce build time, and it was submitted with an estimated build time of approximately 84 hours.

Fabrication of the inner flexible Möbius strip and the flexible gear wheels is somewhat less straightforward. Because we do not have a machine that can directly produce compliant or elastomeric parts using an additive solid free form process, we must introduce yet another process. That process is molding. If we can produce molds for the gear wheels and the inner strip, we can then pour room temperature vulcanizing (RTV) silicone rubber into those molds and release the flexible parts after curing. To address the fact that the inner ring is actually interlocked with the outer ring, we can mold the inner ring as a straight strip, and after it's cured twist it and bond its two ends together using more uncured silicone rubber to form a continuous ring.

Molding, however, introduces yet another hitch into the process. Because SLIDE cannot produce booleans, using slide alone will not allow us to create the negatives necessary for the molds. The solution is to once again introduce Solidworks into the prcocess. The relatively simple geometry of the gear wheel and the straight strip from which the inner Möbius will be made, allows us to import the STL files generated by SLIDE into Solidworks as solid bodies in which each STL triangle is a single face. By doing a boolean subtract operation (using Solidworks' "Cavity" feature) of these bodies from larger blocks we can create the solid models of the molds for these parts.

4 Fabrication

Fabrication of the outer ring is straightforward once the STL file has been sliced acceptably in Quickslice. The file is simply submitted to the FDM machine and left to run until the job in finished.

Fabrication of the flexible components involves just a bit more work. Once the design for the flexible components is completed in Solidworks, the molds are fabricated using a wax deposition 3D printer. Unlike the FDM machine, the time to complete a job on the wax deposition machine is essentially in direct proportion only to the height of the part. Thus, the wax machine can do in a couple of hours a job that might take the FDM machine 16 hours. This choice of the wax machine allows us to complete a design cycle fairly quickly, providing the opportunity to adjust the design of the flexible components easily if needed. The molds for the gear wheels and inner Möbius strip are shown in Figs. 8 and 9.

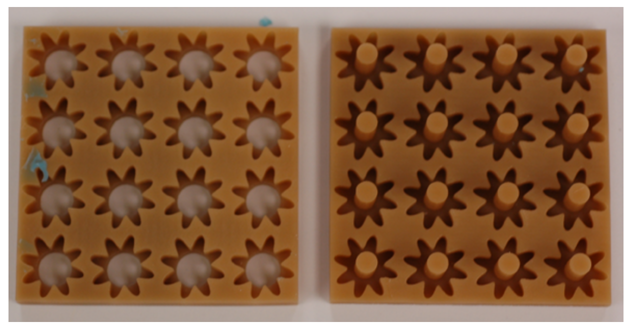

Figure 8: A two part mold for the gear wheels fabricated on the wax deposition machine

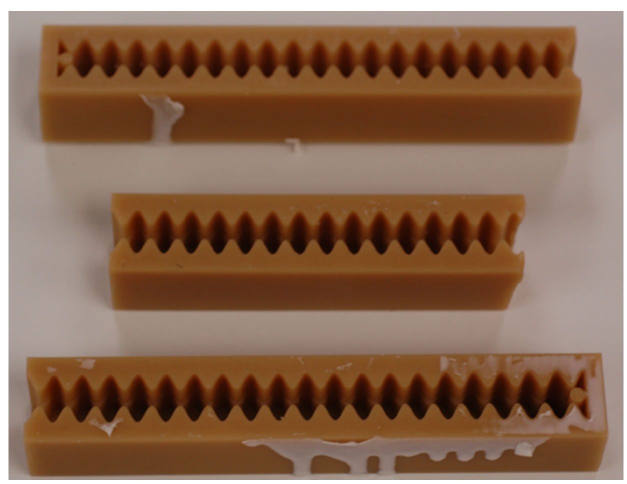

Figure 9: A three piece mold for creating the inner Möbius strip

Once the molds have been fabricated, RTV silicone rubber is mixed, poured into the molds, and allowed to cure

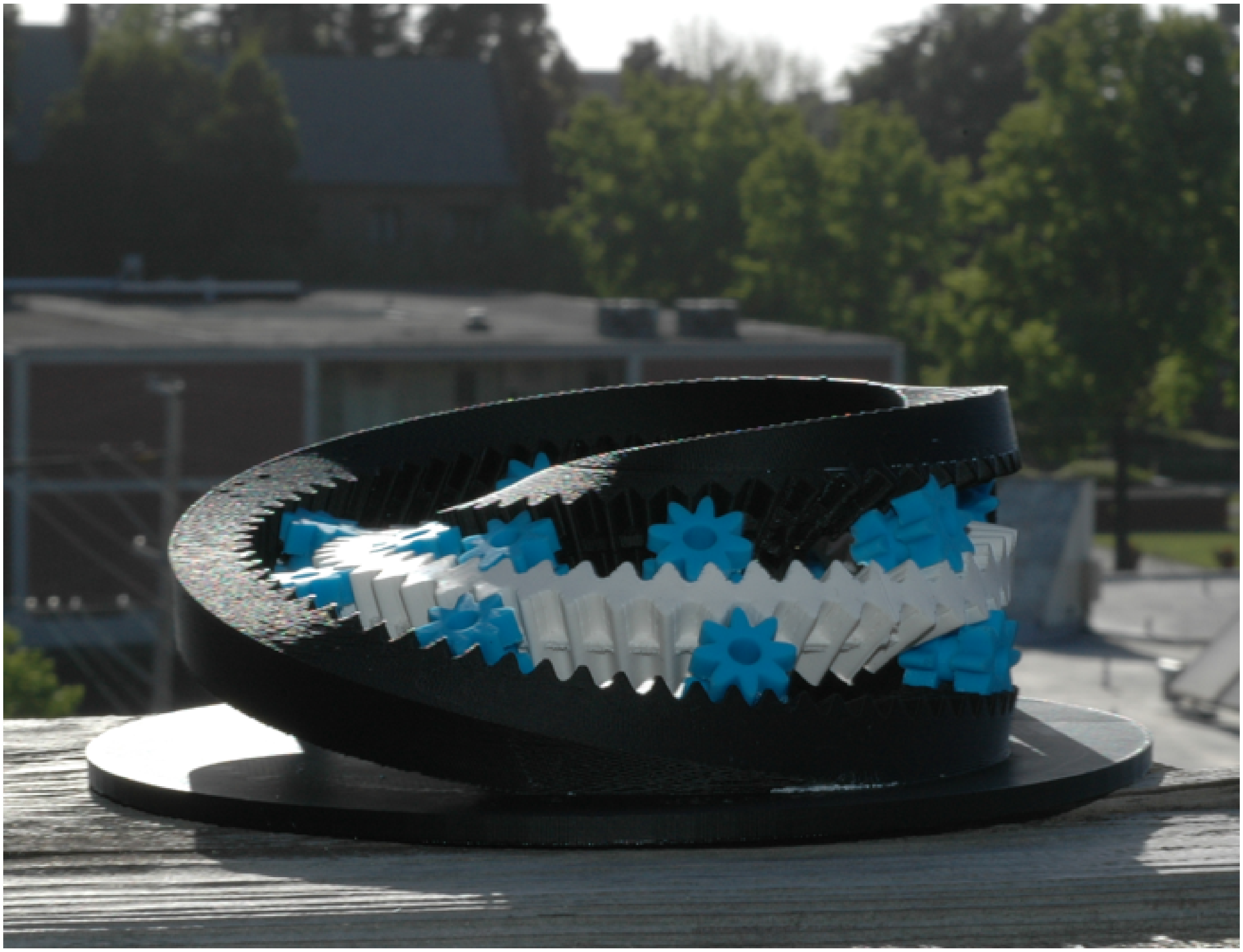

Figure 10: Final model of the Möbius gear

for four to six hours. The inner strip that will comprise the Möbius band is twisted and then interlocked with the rigid outer ring. The ends of the strip are then clamped back together in the center section of the three piece mold and more silicone rubber is poured into the mold to bond the two ends together. The resulting bond is quite strong, and the rubber at the bond appears homogenous, giving the inner Möbius ring a relatively clean look. Lastly the gears are inserted between the face of the inner Möbius strip and the rigid outer ring. The final result is pictured in Fig. 10

5 Lessons Learned and Insights Gained

The most important lesson to be taken from my experience with this project is that no single tool is sufficient to solve a reasonably complicated engineering problem. The likelihood of success (in any problem, really) is heavily dependent on a familiarity with a variety of tools and an understanding of their respective strengths and weaknesses. Procedural modeling is extremely useful in the early stages of design in which we wish to rapidly explore the design space in search of an aesthetically pleasing and mechanically viable solution. The ability to quickly parameterize a solid model and dynamically adjust and update the design in software like SLIDE is invaluable at this early stage. However, when considering fabrication constraints and additional mechanical or structural requirements, a more "engineering feature-oriented" approach found in software like Solidworks is useful. In an ideal world an open standard interchange format that allows users to move between different modeling packages, leveraging the strengths of each to solve complex design problems, would be available and implemented by all major modeling packages. But, until then, it would behoove designers and engineers to maintain a working familiarity with as many tools as possible in order to be able to select the right tool or combination of tools for a given job.